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Abstract: For roughly 15 years, economic research has been involved with chaotic systems. During these years chaos theory took a firm place in science, although the enthusiasm of the first decade was followed by a more subdued kind of consideration. This might be the time to sum up some of the results and to develop some ideas concerning possible applications of chaos theory to economic history (and its theory). Since a good portion of the chaos research that has been done until now deals with financial markets, we will consider that section of economics.

Qualities of chaotic systems and chaotic models of financial markets

Qualities of chaotic systems: Although chaos theory has had a lot of publicity, it seems reasonable to repeat some important qualities of such systems. Other qualities,[1] although also important, are left out because of lack of space. We may divide natural processes into strong deterministic, pure stochastic and dynamic ones. However, dynamic systems themselves can show a certain deterministic or stochastic behavior. Especially the creative processes within a dynamic system are determined by random.[2] In contrast to such stochastic dynamic systems, deterministic dynamic systems can be described by nonlinear and especially recursive differentiable functions.[3] Deterministic chaotic

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[1] Such as self-organization of systems and reversitibilty of processes.

[2] See *Ebeling, Werner,* Zeit und Komplexität: Die kreativen Potenzen des Chaos, in: *Meier, Klaus* u. *Strech, Karl-Heinz* (Hg.), Tohuwabohu: Chaos und Schöpfung. Aufbau, Berlin 1991, S. 79f.

[3] See *Steeb, Willi-Hans,* A Handbook of Terms Used in Chaos and Quantum Chaos.

processes are one class of these systems. That's why nonlinear and recursive functions are a sufficient, but not a necessary condition for the existence of deterministic chaotic systems. Indeed most of the functions used to model financial markets have these forms.

Several paths that lead to chaos have been investigated up to now. The most usual one is via bifurcations. Neighboring solution vectors of a function, called trajectories, usually show similar behavior for a certain length of time. All of a sudden, they move apart. In other words: If a dynamic system depends on a parameter A. and this parameter moves through a critical value, a qualitative change in the behavior of the system occurs. λ is called the bifurcation parameter, the critical value is called the bifurcation point.[4] There are several kinds of bifurcations. The simplest kind passes from an equilibrium into one or more stable or unstable equilibria (local bifurcations, e.g. fold, pitchfork, flip, or transcritical bifurcations).[5] They describe the chaotic phenomenon of period doubling (i.e. the alternation of the system between two modes) in the simplest possible way (see fig. 1). Another frequent way into chaos goes via intermittency. It includes repeated random changes between long regular, laminar phases (called intermissions) and relatively short irregular, deterministic chaotic phases ('windows to chaos').[6] This kind of process is characterized by discrete chaos, which is the opposite of continuous chaos. Among the other ways to chaos, quasi-periodic motion is of particular interest in economics. A motion is quasi-periodic, if it is not periodic, but consists of periodic motions. This might be the case if some of these movements are periodic on a time scale, while other one are periodic on a distance scale. Quasi-periodic motion appears if the periods of these motions do not have a common multiple. Bifurcations might come over a chaotic system without warning, but such a Blue Sky catastrophe is relatively seldom. In financial markets, this kind of bifurcation is known as Noah effect. It takes place after unexpected news concerning namely small caps, foreign exchange (FX) and commodity markets. More often bifurcations are preceded by distinct changes of the system's mode. Such phase transitions can also be found in financial markets: Before stock market crashes the composition and behavior of investors changes and market volatility jerks up.

Deterministic chaotic systems are dissipative ones. Unlike conservative systems, the volume of an element of their phase space tends to zero in the

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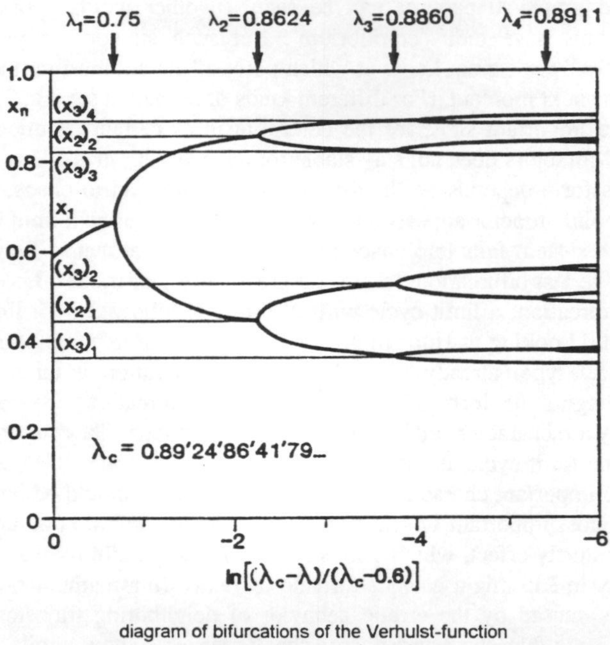
BI-Wissenschaftsverlag, Mannheim etc. 1991, p. 42. These functions usually are sufficient for dynamic systems acting in a more formal way such as financial markets. That's why the discussion in this article usually is done on the basis of deterministic dynamic systems.

[4] **See ibid., p. 25.**

[5] **See *Lorenz, Hans-Walter,* Nonlinear Dynamical Economics and Chaotic Motion. *{Beckmann, M.* and *Krelle, W.* (eds.), Lecture Notes in Economics and Mathematical Systems 334) Springer, Berlin etc. 1989, p. 65-75.**

[6] **See *Steeb* (1991), p. 66f.**



$$x_{n+1} = 4\lambda * x_n * (1 - x_n)$$

Fig. 1: Bifurcations Source: Seifritz (1987), p. 53.

course of time.[7] After this period of time, the mode of the system is characterized by an attractor. An attractor defines the equilibrium level of a deterministic system. It maps the degrees of freedom or orbits of a system to their limit cycles in phase space. There are several kinds of attractors. The simplest one is a point attractor in a system, which can be described perfectly by the help of only two degrees of freedom. Cyclic attractors or limit cycles as the next complicated show the above mentioned skipping of the system between two modes. A further step is to a torus attractor. Such a system has three or more degrees of freedom. A torus attractor is determined by coupling several periodic systems to a system, which is periodic itself. Deterministic chaos is expressed by a strange attractor. The degrees of freedom of such a

[7] **In a phase space, the value of a variable is plotted against possible values of the descriptive variables at the same time. See *Peters, Edgar E.,* Chaos and Order in the Capital Markets. (Wiley finance editions) John Wiley, New York 1991, p. 230.**

system no longer move cyclically, but its determinants could only move on the attractor and graphical patterns can be seen. In other words: Deterministic chaotic systems have their 'equilibrium'. But such an equilibrium is very complicated and we cannot know at which point of the equilibrium the system will be by the next moment. (For different kinds of attractors see fig. **2)** Strange attractors are important signs for the determinism of certain chaotic systems. The form of attractors does not stay stable for each system in each mode of the system. This form depends on the different paths that lead to chaos. In every case first a point attractor appears and after the first bifurcation a limit cycle. At the end, each system falls into chaos showing a strange attractor. Between the second and the last bifurcations, there is a great variety of modes. Looking at a pitchfork bifurcation, a limit cycle with 2^1 limits is followed by a limit cycle with 2^2 limits. Looking at Hopf bifurcations as bifurcations more complicated than the local type, already after the second bifurcation a torus attractor appears, changing its form after two additional bifurcations. Systems with intermittency are characterized by saddlepoint bifurcations. They fall into chaos after only one limit cycle and one intermission

Two more important characteristics of chaotic systems should be mentioned. One of the most important is sensitive dependence on initial conditions. It is called the butterfly effect, which stands for the vague possibility that the wing of a butterfly in Rio might cause a tornado in Texas. In a mathematical sense, this effect is caused by the erratic behavior of neighboring trajectories. The second quality is that characteristics of chaotic systems show similar patterns no matter what local or temporal level is regarded. This scale invariance or self-similarity is one of the most important signs of determinism in chaotic systems. It is true for time series and attractors.[8] This short recapitulation of the qualities of chaotic systems leads in parts to a working definition of chaos. Such systems show

- topically transitivity (from one mode to another e.g. by bifurcations),
- sensitive dependence on initial conditions and
- density of periodic points.

These three objectives mean that chaotic systems cannot be broken down or decomposed into two subsystems (indecomposability), they are are unpredictible, but have an element of regularity.[9] According to another definition, deterministic chaotic systems include

- determinism.

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[8] See *Großmann, Siegfried,* Selbstähnlichkeit: Das Strukturgesetz im und vor dem Chaos, in: *Gerok, Wolfgang* et al. (Hg.), Ordnung und Chaos in der unbelebten und belebten Natur. (Verhandlungen der Gesellschaft Deutscher Naturforscher und Ärzte 115. Versammlung) Hirzel, 2. Aufl., Stuttgart 1990, S. 101-122.

[9] See *Steeb* (1991), p. 32-35.

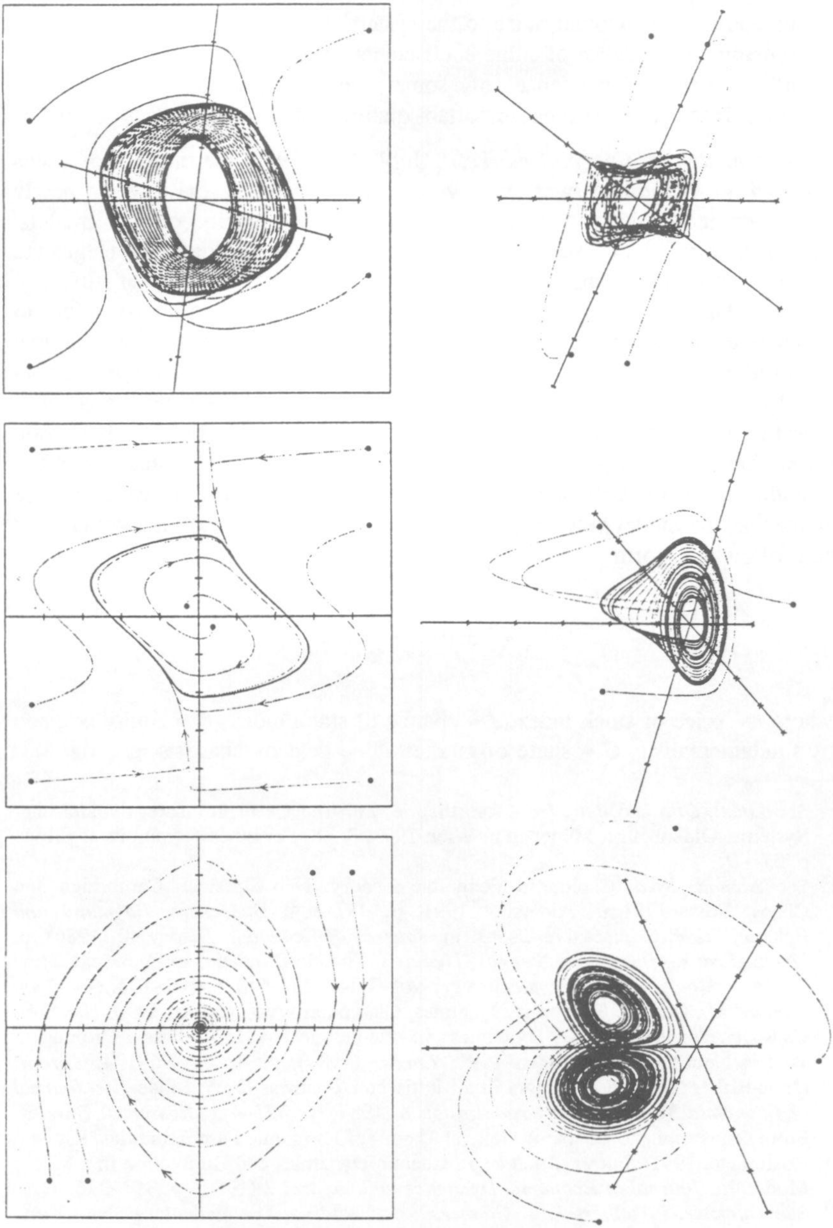

Fig. 2: Attractors Source: Crutchfield (1989), p. 8.

- no addition of external noise to the system,
- sensitive dependence of some coefficients on initial conditions, but
- no sensitive dependence of some global characteristics on these conditions.[10] For the most important qualities of natural systems see fig. 3.

Chaotic models of financial markets: Until about 1988, the purpose of chaos research in economics was mainly to detect chaos in existing or newly developed economic models. These included certain macroeconomic models, capital theory, fields such as urbanism, regional agglomeration, growth frontiers, ecological and economic interdependence.[11] Models of financial markets take a prominent place among them. They mainly refer to organizational or psychological effects. One such effect might be a simple reaction to previous price changes.[12] One of the most common of organizational or psychological effects, however, is the dichotomy between long term investors and speculators and their different investment periods and reaction times. Almost 25 years ago, the English mathematician E.C. Zeeman published a model of the stock market as a cusp catastrophe consisting of exactly these elements. The model puts emphasis to the explanation of stock market crashes and follows the form

$$I = \frac{1}{4} J^4 - F * J - \frac{1}{2} C * J \tag{1}$$

where I - price of stock index, J = change of stock index, F = equity demand by fundamentalists, C - share of market value held by chartists (see fig. 4).[13]

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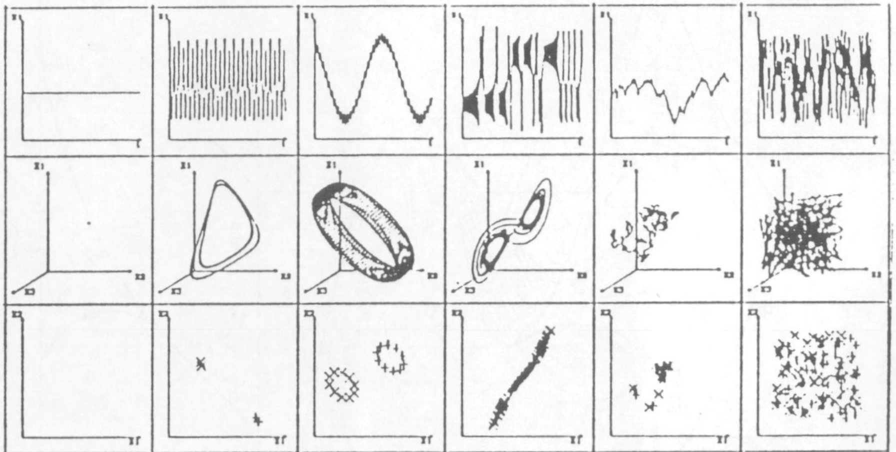
[10] **See *Loistl, Otto* and *Betz, Iro,* Chaostheorie: Zur Theorie nichtlinearer dynamischer Systeme. Oldenbourg, München u. Wien 1993, S. 48. For other definitions see ibid., p. 37-49.**

[11] **See *Ahmad, Syed,* Capital in Economic Theory: Neo-Classical, Cambridge and Chaos. Edward Elgar, Aldershot 1991, p. 337-384; *Deneckere, Raymond and Pelikan, Steve,* Competitive Chaos, in: *Journal of Economic Theory* 40 (1986), p. 13-25; *Frank, Murray* and *Stengos, Thanasis,* Chaotic Dynamics in Economic Time Series, in: *Journal of Economic Surveys* Vol. 2, No. 2 (1988), p. 103-133; *Goodwin, Richard M.,* Chaotic Economic Dynamics. Clarendon Press, Oxford 1990; *Hommes, Carsien Harm,* Chaotic Dynamics in Economic Models. Diss. Groningen, Walters-Noordhoff, Groningen 1991; *Lorenz* (1989), p. 42-61, 96-174; *Lorenz, Hans-Walter,* Strange Attractors in a Multisector Business Cycle Model, in: *Journal of Economic Behavior and Organization* 8 (1987), p. 397-411; *Rosser, J. Barkley,* From Catastrophe to Chaos: A General Theory of Economic Discontinuities. Kluwer, Boston etc. 1991; *Stutzer, Michael J.,* Chaotic Dynamics and Bifurcation in a Macro Model, in: *Journal of Economic Dynamics and Control* 2 (1980), p. 353-376.**

[12] **See *Dockner, Engelbert J.* u. *Gaunersdorfer, Andrea,* Die Bedeutung der Chaostheorie für die empirische Kapitalmarktforschung, in: *Bank-Archiv* Jg. 43, Nr. 6 (1995), S. 428-430.**

[13] **See *Zeeman, E.C,* On the Unstable Behaviour of Stock Exchanges, in: *Journal of Mathematical Economics* 1 (1974), p. 39-49; *Zeeman, E.C:* Catastrophe Theory, in:**



	fixed point	limit cycle	torus	deterministic chaos	random walk (Markoff chains)	random
Hurst exponent	$H=0$	$0 < H < 0.5$	$0 < H < 0.5$	$0 < H < 1$	$0.5 < H < 1$	$H = 0.5$
Lyaponov exponent	$L_1 < 0$	$L_1 = 0$	$L_1 = 0$	$L_1 > 0$	$L_1 > 0$	$L_1 > 0$
correlation dimension	$D = 0$	$D = 1$	$D = 2$	$2 < D < 3$	$2 < D < \infty$	$D = \infty$

Fig. 3: Natural Systems Source: Holzer (1990), p. 156.

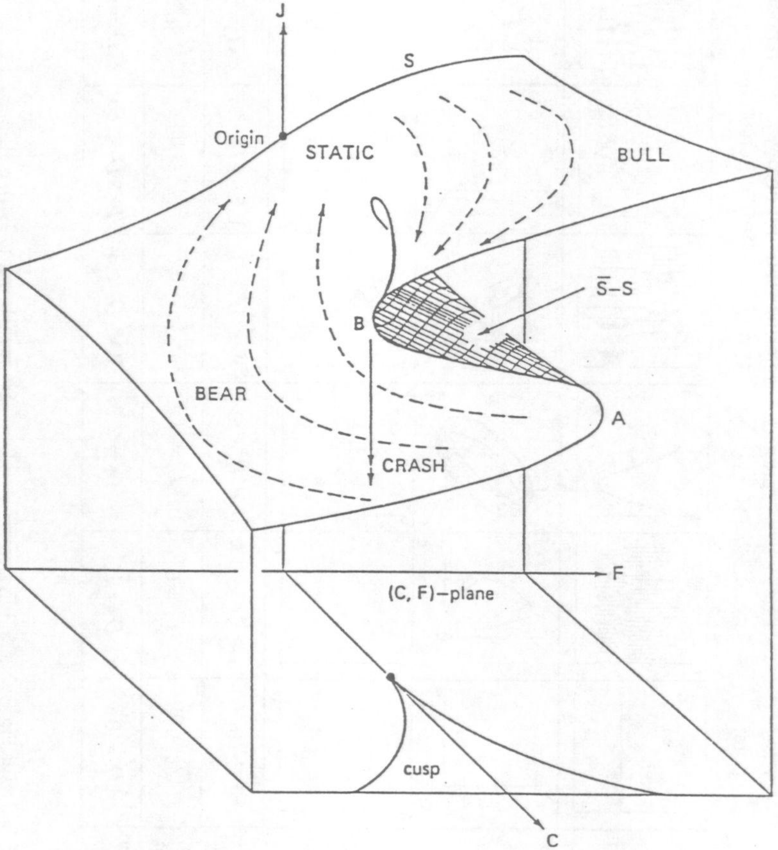


Fig. 4: Stock Market Modell (Cusp Catastrophe) Source: Zeeman (1974), p.368.

Out of E.C. Zeeman's model, E.E. Peters developed his *Fractal Market Hypothesis.* He points out that differences in investment horizons are due to market instabilities, while during normal times fractal structure gives the market considerable stability.[14] T. Vaga published his *Coherent Market Hypothesis* as a nonlinear statistical model. He distinguishes four market phases: random walk, transition, chaotic markets, and coherent markets. Each one is characterized by different kinds of attitudes and the mutual influence of investors. The model follows the psychological theory of social imitation, but is formulated mathematically.[15] A recent attempt to determine the behavior of financial market operators by a model was done by Lux. He called his paper 'Socio-Economic Dynamics in Speculative Markets'. Lux also distinguishes between the two interacting groups of speculators and investors, by using the literature of crowd psychology and synergetics. One of the main aims of his work is to show that his model creates both leptokurtosis[16] and chaos.[17] Within the last decade, additional attempts have been made to model financial markets with the help of different feed back loops which imitate the behavior of certain market participants in certain situations.[18] Many of these loops follow stock exchange savings which in some cases can be traced back to the beginnings of early stock markets. These models, although not consistent until now, might give hints for reconstructing the behavior of historical financial markets. Maurice R. Larrain developed one of the few chaotic models of security markets based on fundamental determinants. His K-Z interest rate model combines Keynesian economics with past interest rates.[19] Sherrill Shaffer published a model which offered proof that apart from trading behavior simple fundamental causes such as a fixed dividend payout ratio combined with a declining marginal efficiency of the investment curve may suffice to produce chaos, if some relevant parameters show certain values.[20]

Scientific American **Vol. 234 (1976), p. 65-83. Symbols here and in the following are adjusted. For further applications of catastrophe theory to economics see *Ursprung, Heinrich W.,* Die elementare Katastrophentheorie: Eine Darstellung aus Sicht der Ökonomie. *(Beckmann, M.* and *Künzi, H.P.* (Hg.), Lecture Notes in Economics and Mathematical Systems 195) Springer, Berlin etc. 1982.**

[14] See *Peters, Edgar E.,* Fractal Market Analysis: Applying Chaos Theory to Investment and Analysis. (Wiley finance editions) John Wiley, New York 1994.

[15] See *Vaga, Tonis,* The Coherent Market Hypothesis, in: *Financial Analysts Journal* Vol 46 (1990), p. 36-49.

[16] A frequency distribution having "fat tails'.

[17] See *Lux, Thomas,* The Socio-Economic Dynamics of Speculative Markets: Interacting Agents, Chaos, and the Fat Tails of Return Distributions. Presentation held at the annual meeting of "Verein für Socialpolitik', Sept. 21st, 1995 in Linz.

[18] See *Tvede, Lars,* Psychologie des Börsenhandels. Gabler, Wiesbaden 1991, S. 130-141, 304-317.

[19] See *Peters* (1991), p. 187-191.

[20] See *Shaffer, Sherrill,* Structural shifts and the volatility of chaotic markets, in: *Journal of Economic Behavior and Organization* 15 (1991), p. 204-209.

Measures of nonlinearity and chaos

To investigate time series of historical financial markets, the usual first step is to find out if the data are identical and independently distributed (IID). Independent variables are required by almost all popular models of securities' returns, and many of them also presume identical distributions.[21] By proving IID, researchers try to answer the question, whether certain time series or their changes (Markoff chains) follow a random walk or else a deterministic path.[22] This research in the first step aims to answer the question if certain financial markets are efficient. Both independence of variables and identical distributions can be tested out by a broad variety of instruments. Among them are measures testing the distributions for leptokurtosis or for conditional heteroskedasticity[23], e.g. skewness - qualities which are characteristic for time series of financial markets. In this context, only widely used ARCH (autoregressive conditional Heteroskedasticity), GARCH (generalized ARCH) and EGARCH (exponential GARCH) models,[24] or the Kiefer-Salmon test for normal kurtosis and normal skewness[25] should be mentioned. In some text books *spectral analysis* is described as a method for identifying nonlinearities and chaos. Although this instrument is particularly useful for distinguishing between random behavior and periodic time series with only a few frequencies, it cannot distinguish between chaotic and true random behavior. Nevertheless, some authors point out that it is possible to isolate significant chaotic peaks with the help of the spectrogram belonging to the method, because they think that a broad band spectrum 'is, in practice, a reliable indicator of chaos'.[26]

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[21] See *Akgiray, Vedat,* Conditional Heteroscedasticity in Time Series of Stock Returns: Evidence and Forecasts, in: *Journal of Business* vol. 62, no. 1 (1989), p. 60.

[22] Theoretically there are certain other possibilities, e.g. strong deterministic or stochastic nonlinear connections.

[23] Uneven dispersion, having 2 important consequences for the estimation: (1) The least squares estimators of the regression coefficients are no longer efficient or even asymptotically efficient. (2) The estimated variances of these estimators are in general biased.

[24] See for ARCH models *Engle, R.F.,* Autoregressive conditional heteroscedasticity with estimates of the variance of U.K. inflations, in: *Econometrica* 59 (1982), p. 987-1007; for GARCH models *Bollerslev, T.,* Generalized autoregressive conditional heteroskedasticity. in: *Journal of Econometrics* 31 (1986), p. 307-327; for EGARCH models *Nelson, D.,* Conditional heteroskedasticity in asset returns: a new approach, in: *Econometrica* 59 (1991), p. 347-370. For a summary of ARCH models see *Bollerslev, T., Chou, R.Y.* and *Kroner, K.F.,* ARCH modeling in finance. A review of the theory and empirical evidence, in: *Journal of Econometrics* 52 (1992), p. 5-59.

[25] See *Kiefer, N.* and *Salmon, M.,* Testing normality in econometric models, in: *Economics Letters* 11 (1983), p. 123-127.

[26] *Baker, E.L. and Gollub, J. P.,* Chaotic Dynamics. Cambridge University Press, Cambridge 1990, p. 61. See for spectral analysis and its application for detecting chaos in time series: *Loistl/Betz* (1993), S. 50-56; *Lorenz* (1991), p. 176-179; *Medio, Alfredo,* Chaotic Dynamics: Theory and Application to Economics. Cambridge University Press, Cambridge 1992, p. 101-114.

Capital market theory developed special tests for the efficiency of financial markets, e.g. the proof of absence of excess volatility.[27] Some new instruments answer the same question and usually even do a step more. One of them is the *Hurst exponent* as a measure of the kind of mathematical process a time series follows. Calculating this exponent is part of the Rescaled Range or R/S analysis. In an otherwise quiet system the range R of a spot's motion during a period *m,* rescaled by its standard deviation *S,* follows the equation

$$\frac{R_m}{S_m} = c * m^H \tag{2}$$

for c = constant. *H* is the Hurst exponent

$$H = \frac{\log\left(\frac{R_m}{S_m}\right) - \log(c)}{\log(m)}. \tag{3}$$

It may take on values between 0 and 1. A value of 0.5 is typical for a random walk. A Hurst exponent different from 0.5 means that a time series' changes are not normally distributed. Significantly different values show evidence for deterministic structure in the data. $H > 0.5$ indicates the presence of a persistent time series. The more *H* approaches 1.0, the stronger the system's trend-reinforcing behavior gets. In addition, high Hurst values show less noise and clearer trends than lower ones.[28] In the case of $H < 0.5$ antipersistent connections between subsequent spots are likely. As the Hurst exponent relies on rescaled data with an average of 0 and a standard deviation of 1, the method is scale invariant. It therefore allows one to compare data from different time periods and different time scales.[29] Not only the behavior of the system can be determined from R/S analysis, but also the length of its cycles, which is the period of time in which a system continuously shows persistent or antipersistent behavior.[30] This gives a possibility for determining long memory cycles.

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[27] See *Joerding, Wayne,* Are Stock Prices Excessively Sensitive to Current Information? in: *Journal of Economic Behavior and Organization* 9 (1988), p. 71-85. For an application of this method for historical financial markets see *Delong, J. Bredford* and *Becht, Marco,* "Excess volatility" and the German stock market, 1876-1990. (EUI working paper ECO No. 92/82) Badia Fiesolana, San Domenico (FI) 1992. Recently, the informative value of this proof has been doubted. See *Krämer, Walter,* A note on excess volatilities in empirical capital market research, in: *Zeitschrift für Wirtschafts- und Sozialwissenschaften* 2 (1994), p. 173-183.

[28] See *Peters* (1991), p. 89.

[29] Persistent or trend-reinforcing series. See ibid., p. 65.

[30] See ibid., p. 62-77.

The *fractal dimension D* is another method for distinguishing between stochastic and deterministic systems. *D* shows to what extent the attractor of a system fills up its phase space. It can be regarded as the inverse of the Hurst exponent $D = 1/H$. There is a whole family of fractal dimensions to quantify certain characteristics of attractors. Their values take on integers *0, 1, ..., n* in the case of not-chaotic attractors like points, limit cycles and n-tori. Time series following a random walk have $D - 2$ in a phase space, which can be reconstructed by the map $x_t \rightarrow x_{t+1}$. Such a stochastic time series fills up its phase space completely, while deterministic generated time series move exclusively on their attractors. The fractal dimension could also be used as a risk measure. Time series following a consistent trend have lower fractal dimensions than time series following a random walk. Unlike measures of dispersion such as the standard deviation, the fractal dimension shows a time path and is an interesting alternative for measuring risk of change from an actual mode.[31] The fractal dimension shows the maximum number of degrees of freedom or of determinants of a dynamic system

To determine the fractal dimension one has to classify a phase space with an embedding dimensional dim_E into cells V, with a volume of R^{dim_E} and an edge length of ε. $N(\varepsilon)$ is the minimum number of cells necessary to cover the attractor. If the cells V_i are numbered from $i = 1$ until $N(\varepsilon)$ and the probability of finding a point of an attractor in cell V_i is called p_i, the fractal dimension is defined as

(4)
$$D_q = \lim_{\varepsilon \to 0} \frac{1}{q-1} * \frac{\log\left(\sum_{i=1}^{N(\varepsilon)} p_{iq}^{q}\right)}{\log(\varepsilon)}$$

with $D_q \geq D_p$ for $q \leq p$.[32]

For several *q* one gets special measures e.g. for $q = 0$ the Hausdorff dimension, for $q \rightarrow 1$ the information dimension and for $q = 2$ the correlation dimension. All these dimensions are called fractal dimensions.[33] For dimensions $D > 2$ the fractal dimension can be approximated by the Grassberger-Procaccia algorithm:

(5)
$$D_2 = \lim_{\varepsilon \to 0} \left[\frac{\log C(\varepsilon)}{\log(\varepsilon)}\right]$$

[31] See **ibid.,** p. 59f.
[32] **See *LoistlIBetz*** (1993), S. 80-83; ***Büzug, Thorsten,*** **Analyse chaotischer Systeme.** BI-**Wissenschaftsverlag, Mannheim etc.** 1994, **S.** 56-59.
[33] **See *Steeb*** (1991), p. 51, 31, 55f., 65f.

where C(ε) is the correlation integral. This integral measures the likelihood that the distance between two neighboring spots on trajectories is less than ε. The exact definition of the correlation integral is

$$C(\varepsilon) = \lim_{N \to \infty} \frac{1}{N^2} * \sum_{\substack{i,j=1 \\ i \neq j}}^{N} \Theta * \left(\varepsilon - \left\| x_i - x_j \right\| \right) \tag{6}$$

where

$$N = N_{dat} - \left(\dim_E - 1 \right) * \frac{\tau}{T_a} \tag{7}$$

the number of points in the embedding space (N_{dat} = number of data points of time series; τ - time of delay; T_a - observation period) and x_i, x_j = independent vectors in the embedding space, Θ stands for the Heaviside function[34]

$$\Theta\left(x\right) = \begin{cases} 1 \textit{ für } x > 0 \\ 0 \textit{ für } x \leq 0 \end{cases}. \tag{8}$$

The Grassberger-Procaccia algorithm works well with long time series of good quality. Problems may occur with noisy data[35] and systems following intermittency.[36]

There has been a broad discussion about the number of data points necessary to calculate the correlation dimension accurately. The relatively optimistic opinion of Eckmann and Ruelle is that with a Grassberger-Procaccia algorithm no higher fractal dimension could be measured than

$$D_2^{\max} = \frac{2 * \log(N)}{\log\left(\frac{1}{\varepsilon}\right)}. \tag{9}$$

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[34] See *Buzug* (1994), S. 56-60; *Loistl/Betz* (1993), S. 80-83.

[35] For the filtering of time series see *Buzug* (1994), S. 136-152. For the importance of noise for trading on stock markets see *Heyl, Daniel C. Freiherr* v., Noise als finanzwirtschaftliches Phänomen: Eine theoretische Untersuchung der Bedeutung von Noise am Aktienmarkt. (Schriftenreihe des Instituts für Kapitalmarktforschung an der Universität Frankfurt/M. Bd. XVI) Diss. Frankfurt/M., Knapp, Frankfurt/M. 1995.

[36] See *Ruelle, David*, Deterministic Chaos: The Science and Fiction. (Proceedings of the Royal Society, London) 427 A (1990), p. 241-248; *Grassberger, P.* and *Procaccia, J.*, Measuring the strangeness of strange attractors. in: *Physica* D9 (1983), p. 189-208.

According to them, for scaling intervals $0.005 \leq \varepsilon \leq 0.1$ and a fractal dimension of $D_2 \approx 7$ $N_{Dat} \approx 33.000$ data points are needed.[37]

The *BDS statistic,* developed by Brock, Dechert and Scheinkman, tests the null hypothesis that the data are independently and identically distributed. The method is based on the above correlation integral (6). Let $\{x_t : t = 1, ..., T\}$ be a sequence of IID observations. Form N-dimensional vectors $x_t^N = (x_{t+p}, ...,$ ..., $x_{t+n-1})$, called N-histories. The correlation integral becomes

$$(10) \qquad C_N(\varepsilon) = \frac{2}{T_N * (T_N - 1)} * \sum_{1 \leq i < j \leq T_n} \prod_{k=0}^{m-1} \Theta * \left(\varepsilon - \left\| x_{i+k} - x_{j+k} \right\| \right).$$

Under the null hypothesis of an asymptotic standard normal deviation we get the BDS statistics

$$(11) \qquad bds = w_N(\varepsilon) = \frac{\sqrt{T} * \left[C_N(\varepsilon) - C_1(\varepsilon)^N \right]}{\sigma_N(\varepsilon)}$$

A rejection of the null hypothesis points out that there is some type of dependence in the data, resulting either from a linear or nonlinear stochastic system, or a nonlinear detenninistic system. To identify the type of system, further research is necessary. W. A. Brock showed that a (strong or chaotic) deterministic time series y_t has deterministic (chaotic) residuals μ_t which can be calculated by the regression[38]

$$(12) \qquad y_t = \sum_{i=1}^{I} \alpha_i * y_{t-1} + \mu_t \, .$$

y_t and μ_t both have the same correlation dimension and the same Lyapunov exponent (see later). On the basis of this theorem both linear and nonlinear models can be adapted to the analyzed data. The resulting residuals may be analyzed by methods of chaos theory. BDS statistics can be applied as a measure for systematic nonlinearities in ARCH, GARCH, and EGARCH models. Knowing this, the BDS test builds an interesting bridge between chaos theory and economic modeling, although at this point specific research still has to be done.[39]

"See *Buzug* (1994), S. 59.

[38] See *LoistllBetz* (1993), S. 104; *Brock, William,* Distinguishing random and deterministic systems: Abridged version, in: *Journal of Economic Theory* 40 (1986), p. 168ff.

One of the most important measures for a system's current chaos is the *Lyapunov exponent L.* It measures the divergence of two neighboring trajectories after *t* periods. *L* is therefore a measure for the predictability of a system. It has to be calculated for every time $\leq T$ and every dimension *D.* The maximum Lyapunov exponent calculated determines the behavior of the system. If *D* is not known, *L* has to be approximated.

From an empirical time series $X = (x_0, x_1, x_2, \ldots, x_T)$ m-dimensional phase spaces z are formed:

$$z_t = \left(x_t, x_{t+1}, x_{t+2}, \ldots, x_{t+m+1}\right) \text{ with } t = 1,2,\ldots,T-m+1. \tag{13}$$

Because of this, *T-m + 1* plots in a wi-dimensional phase space can be found. For all neighboring spots (a_j, a_k), for which is true $|a_j - a_k| < \varepsilon$ with $a_j \neq a_k$. In a next step, for these *N* pairs of neighboring spots, the distance δ after *p* periods can be calculated as

$$\delta_p^{(j,k)} = \frac{\left|a_{j+p} - a_{k+p}\right|}{\left|a_j - a_k\right|}. \tag{14}$$

Then the Lyapunov exponent follows the function

$$\lambda = \frac{1}{p * N} * \sum_{j,k} (\ln \delta_p^{(j,k)}). \tag{15}$$

Negative *Ls* show a contraction in phase space. That means that the distance between two spots shrinks in the course of time. After disturbance such a system will return to a stable attractor. Positive Ls describe a dispersion in phase space. The more *L* grows, the more sensitively the system reacts to the change of its starting conditions. (See fig. 5 for the growth of the Lyapunov exponent depending on the growth of the bifurcation parameter.) Although deterministic, the system becomes unpredictable after certain periods of time.

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[39] **For BDS statistics see *Brock, William A., Dechert, W.D.* and *Scheinkman, José A.,* A test for independence based on the correlation dimension. (SSRI working paper no. 8702, Dept. of Economics, University of Wisconsin) Madison 1987; *Brock, William A., Hsieh, David A.* and *LeBaron, Blake,* Nonlinear Dynamics, Chaos, and Instability: Statistical Theory and Economic Evidence. MIT Press, Cambridge, Mass., and London 1991, p. 41-81; *LoistllBetz* (1993), S. 102-104. For its application in ARCH models see *Engle* (1982), p. 987ff.; in GARCH models see *Bollerslev* (1986), p. 307ff.; in EGARCH models see *Nelson* (1991), p. 347ff. For the enlarged application of the BDS test see *LoistllBetz* (1993), S. 102ff.**

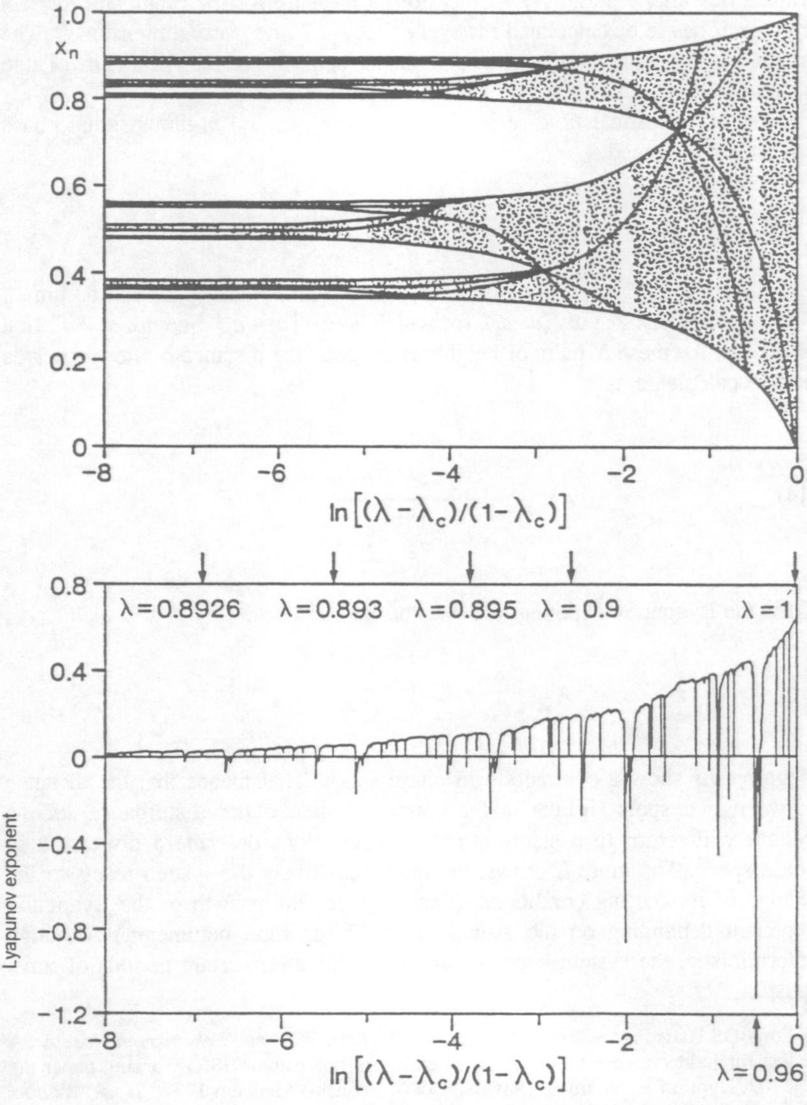

Fig. 5: Lyapunov-Exponent Source: Seifritz (1987), p. 62.

This loss of explicable quality depending on the number of iterations is expressed in *L.* After 1/L periods of time no information at all about starting conditions can be found. That's why calculating 1/L is another way to determine the period of a long memory cycle.

A further important measure which can characterize chaotic movement in an n-dimensional phase space is the *Kolmogorov entropy K.* It is based on the above mentioned information dimension: If the phase space is divided up into N boxes, there is a probability

(16) $$p_i(\varepsilon) = \frac{N_i(\varepsilon)}{N(\varepsilon)}$$

for the mean stay of a trajectory in the box with the number *i.* $p_i(\varepsilon)$ is the number of data points in box $N(\varepsilon)$ divided by the number of boxes $N(\varepsilon)$ that are not empty. According to the mathematical information theory[40] the lacking information to locate the system's mode can be written with a given precision by Shannon's information measure:[41]

(17) $$I(\varepsilon) = -\sum_{i=1}^{N(\varepsilon)} p_i(\varepsilon) * \log p_i(\varepsilon).$$

To define Kolmogorov entropy *K,* the probability p_i is replaced by the linked probability $p_{i_0,\dots,i_m}$, which determines that the system at time *t* stays in box I_0, at time $t + \Delta t$ in box i_1 and at time $t + m\Delta t$ in box i_m. The information needed to locate the system's mode can be defined in a way similar to Shannon as

(18) $$K_m = -\sum_{i_0,\dots,i_m} p_{i_0,\dots,i_m} * \log p_{i_0,\dots,j_m}.$$

The additional information for predicting in which box i^*_{m+1} the system can be located after the next evolutionary step K_{m+1} can be written as $K_{m+1} - K_m$. In other words *K* measures the loss of information of the system developing from *m* to ***m+1.*** The Kolmogorov entropy *K* therefore can be written as the mean rate of information losses of a dynamic system:

(19) $$K = \frac{1}{n} * \sum_{m=1}^{n} \left(K_{m+1} - K_m\right)$$

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[40] **See *Grosche, G.* et al. (Hg.): Teubner-Taschenbuch der Mathematik (Bronstein/ Semendjajew), Teil II. 7. Aufl., Teubner, Stuttgart u. Leipzig 1995, S. 51lf.**
[41] **See *LoistllBetz* (1993), S. 85f.**

Inserting Shannon's information measure and constructing limiting values, the expanded Kolmogorov entropy of order q can be determined. It is written as

$$(20) \qquad K_q = -\lim_{\varepsilon \to 0} \lim_{m \to \infty} \left(\frac{1}{m * \Delta t} * \log C_m^q(\varepsilon) \right)$$

with the correlation integral

$$(21) \qquad C_m^q(\varepsilon) = \left\{ \frac{1}{N} * \sum_i \left[\frac{1}{N} \sum \Theta * \left(\varepsilon - \left(\sum_{k=0}^{m-1} \left(y_{i+k} - y_{j+k} \right)^2 \right)^{\frac{1}{2}} \right) \right]^{q-1} \right\}^{\frac{1}{q-1}}$$

K is defined as $0 \leq K \leq \infty$. If the system is strong deterministic, K is 0 , if it behaves in a pure stochastic manner, it is ∞. If the system is chaotic, K is positive and finite. K can be used to determine the interval T for which predictions about the mode of the chaotic system are possible. T follows the form

$$(22) \qquad T = \frac{1}{K} * \log \left(\frac{1}{\varepsilon} \right).$$

In other words the accuracy of measure ε influences the interval T only in a logarithmic manner. So if the prediction interval should be doubled, the accuracy of the measure must be squared. Chaos researchers usually suppose that there is a relationship called Ruelle's relation between the Kolmogorov entropy K and a positive Lyapunov exponent L:[42]

$$(23) \qquad K \leq \sum_i \lambda_i^+ \text{ with } \lambda_i \geq 0 .$$

This short and necessarily incomplete description of some of the most important measures for detecting nonlinearities and chaotic behavior is presented as an introduction. The methods are at present under constant discussion. Nevertheless, this should be sufficient to get an idea about the instruments chaos researchers are using.

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[42] **See ibid., S. 89f., 95-100; *Seifritz* (1987), S. 64f.**

Research on nonlinearity and chaos with empirical time series in financial history

Within the last years the emphasis of research shifted from detecting chaos in economic models to the investigation of empirical time series. As most instruments of chaos theory require a very large sample size, these studies cover at least a decade and thus have an historical aspect. Although some examine time series of business cycle indicators, most of them deal with financial markets. This has several reasons:

- Financial markets offer much higher numbers of data points than other economic fields.
- The available data are often of superior quality e.g. they are more precise, at least if they are not the result of broad aggregations as in the case of financial indices.
- Many data of financial markets show considerable constancy during the course of time. Time series continuously have a similar quality.
- Some research points to a low number of determinants for certain financial markets. This would mean both a low complexity of the systems and a relatively steady behavior.

The following chapter will give a short survey on research done on chaotic behavior of financial markets. Most of this research concerns with the existence of nonlinearities. This research has been published in an extraordinarily broad range of scientific journals in very different fields such as mathematics, physics, economics, and business administration. This is one of the reasons why this chapter can only give hints about the development. Other reasons are limited space in this article and the author's intention to show only the outlines of the development

Discovering nonlinearities: 1989 Scheinkman/LeBaron analyzed 5.200 daily stock returns of the US market. The data came from the Center for Research in Security Prices at the University of Chicago (CRSP) and consisted of a value-weighted, dividend adjusted index from July 1962 until December 1985. Scheinkman/LeBaron applied several methods such as the BDS-test, the ARCH model and the Grassberger-Procaccia algorithm. The authors discovered market imperfections and the absence of randomness in price changes. Especially in the weekly averages of the data nonlinearity could be proved.[43] At the same time, Akgiray analyzed 6.030 daily stock returns of about the same sample (Jan. 1963- Dec. 1986). Using several statistical instruments including the Kiefer-Salmon test and the maximum log-likelihood function, he also found nonlinearities such as skewness and leptokurtosis in the whole sample and in

[43] **See *Scheinkman, José A.* and *LeBaron, Blake,* Nonlinear Dynamics and Stock Returns, in: *Journal of Business* vol. 62, no. 3 (1989), p. 311-337, esp. p. 319-334.**

each of its five-year periods. ARCH and GARCH models fitted very well to the data giving strong evidence for the thesis of conditional heteroskedasticity.[44] Brock/Hsieh/LeBaron published another analysis of CRSP data two years later. They used the same sample as Scheinkman/LeBaron. Applying BDS statistics, the authors rejected IID. Tsay and Engle statistics showed the nonlinear structure in the data and (G) ARCH models again fitted exactly.[43] The authors also analyzed Standard & Poor's 500 weekly return series from 1928 to 1985.[46]. Brock/Hsieh/LeBaron divided up their data into several subperiods, dropping the 1940s due to World War II. Although the S&P 500 stock market index does not include dividends, again all tests (e.g. BDS and (G)ARCH types of tests) rejected IID.[47]

An even stronger rejection of the random walk character of the data was stated by the authors for several foreign exchange markets. They analyzed each of the 2.510 observations of daily closing bid prices of the US $ versus ¥, DM, £, Can $ and sFr from January 2nd 1974 to December 30th, 1983. To test whether the random walk hypothesis of price changes on efficient markets holds, rates of change have been calculated from the data by taking the logarithmic differences between successive trading days. Once again, the BDS test and (G)ARCH models have been applied. They show that daily exchange rate changes are not independent of past changes and therefore reject random walk hypothesis.[48] ARCH and GARCH models have been applied to FX markets by a number of other authors, too.[49] Hsieh analyzed the same series of observations two years earlier by several measures for nonlinearity and found leptokurtosis. BDS statistics and autocorrelation of the squared data indicated substantial nonlinear dependence.[50] In recent times, a new generation of research concerned interest rate structure, its behavior and determinants.[51] Research about nonlinearities in the term structure has been done by Pfann/Schotman/Tschemig. They compared monthly data of 3-month t-bill rates and 10-year government rates of the US market The data showed extremely high kurtosis of the first differences. This was partly due to

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[44] **See *Akgiray* (1989), p. 55-80, esp. p. 58-66 and 74-79.**

[45] **See *Brock/Hsieh/LeBaron* (1991) p. 95-98.**

[46] **All three publications mentioned until now prefer weekly averages because the daily ones were more noisy. This preference is based on the Scheinkmann/LeBaron suggestion that the weekend interrupts continouity of trading. See *Scheinhnanl LeBaron* (1989), p. 317.**

[47] **See *Brock/Hsieh/LeBaron* (1991), p. 99-101.**

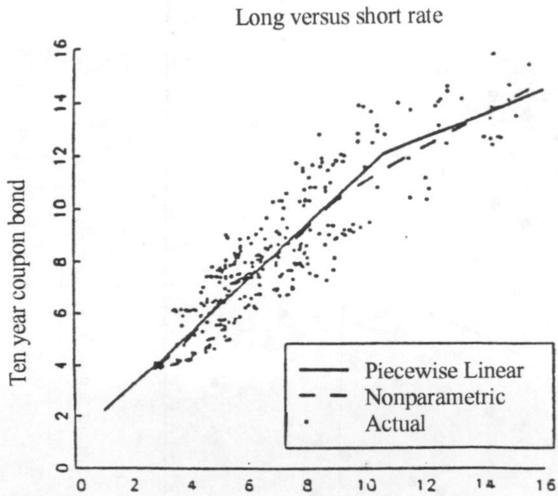
[48] **See ibid., p. 130-145.**

[49] **See ibid., p. 139f.**

[50] **See *Hsieh, David A.,* Testing for Nonlinear Dependence in Daily Foreign Exchange Rates, in: *Journal of Business* vol. 62, no. 3 (1989), p. 339-368, esp. p. 345-367.**

[51] **See *Granger, C.WJ.,* Modelling Non-Linear Relationship between Long-Memory Variables. (Working Paper, University of California) San Diego 1993; *Anderson, H.M.,* Transaction Costs and Nonlinear Adjustment towards Equilibrium in the US Treasury Bill Market. (Working Paper, University of Texas) Austin 1994.**

heteroskedasticity, which itself is consistent with the fact that volatility of interest rates is positively related to the interest rate level. Applying a modified version of the self-existing threshold autoregressive (SETAR) model to the data, the authors discovered two modes influencing US interest rate structure. Until money market rates reach double digits, the data randomize. Beyond this level, they show a mean reverting tendency showing slight nonlinear dynamics. This means that the actual long rates are not well predicted at the extreme values of the short rate (see fig. 6).[32]



Three month T-bill rate

Cross sectional relation between long and short rate

Fig. 6: Interest Rate Structure **Source: Pfann/Schotman/Tsciiernig (1994), p. 21.**

Detecting chaos: Discovering nonlinearities in long running time series of financial markets could only be the first step. Discovering chaotic behavior should follow. As a matter of fact this research was been intensified at the beginning of the 1990s. Nevertheless some work was published previously. The most renown and by far the earliest example is Mandelbrot's analysis of cotton prices for scale invariance. He had a threefold data basis including daily closing

[52] **For the first publication of this paper see *Pfann, Gerhard, Schotman, Peter* and *Tschernig, Rolf,* Nonlinear Interest Rate Dynamics and Implications for the Term Structure. Q^iscussion Paper 43, Sonderforschungsbereich 373, Humboldt-Universität) Berlin 1994.**

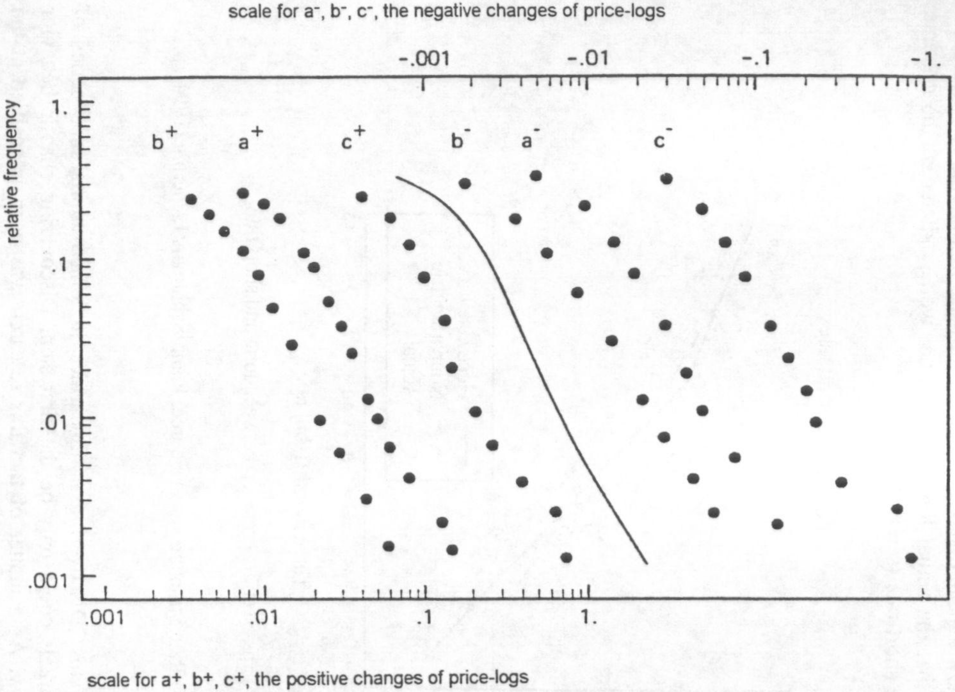


Fig. 7: Scale Invariance Source: Mandelbrot (1991), p. 356.

prices in New York between 1900 and 1905 and at several US places between 1944 and 1958, and New York mid-month closing prices between 1880 and 1940. The results were published first in 1963.[53] Later on, Mandelbrot demonstrated the scale invariance of many other commodity prices, some interest rates and some 19th century stock and bond prices (see fig. 7).[54] Fama and Roll followed by analyzing more recent security prices and some other kinds of interest rates.[55] The results allow several conclusions:

- There is no random walk, since all usual rules of Brown's movement contain privileged time scales.
- A scale invariance has been found in price changes. This scaling law determines price changes which are independent of external shocks such as depressions etc.
- It has been proved stationary over extremely long periods.[56]

Relatively early, but much later than Mandelbrot's articles, research was done by Scheinkman/LeBaron (1986). They analyzed 5.000 daily stock returns from the 1960s up to early 1980s and found slim evidence for a nonlinear dynamic structure, a positive Lyapunov exponent, a fractal dimension between 5 and 6, an imbedding dimension m = 14 and thus mean orbital periods of about 4 years. Their results are consistent with chaos.[57] Later on, this research was criticized because of its inadequate data basis. Peters thinks that for the methods applied more than 10.000 data points would have been needed.[58] Early works also were published by Frank/Stengos, who examined gold and silver rates of return. They found a dimensionality between 6 and 7 and a Kolmogorov entropy around 0.2. This mean an imbedding dimension of roughly 25, referring to a tent map system with 3 to 6 dimensions. Applying Brock's residual test, they found out that residuals from a linear or smooth nonlinear transformation of the data yielded the same correlation dimension as the original data. This means that the series follow deterministic chaos.[59] In the early 1990s, a greater variety

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[53] See *Mandelbrot, Benoît B.,* Variance of Certain Speculative Prices, in: *Journal of Business* 36 (1963), p. 394-419.

[54] See *Mandelbrot, Benoît B.,* Sporadic Random Functions and Conditional Spectral Analysis; Self-Similar Examples and Limits, in: *LeCam, L.* and *Neyman* (eds.), Proceedings of the Fifth Berkeley Symposion on Mathematical Statistics and Probability 3 (1967), p. 155-179.

[55] See *Fama, Eugene F.,* Mandelbrot and the steady Paretian hypothesis, in: *Journal of Business* 38 (1963), p. 34-105; *Roll, R.,* Behavior of Interest Rates: the Application of the Efficient Market Model to U.S. Treasury Bills. Basic Books, New York 1970.

[56] See *Mandelbrot, Benoît B.,* Die fraktale Geometrie der Natur. Birkhäuser, Basel etc. 1991, p. 353-356.

[57] See an unpublished paper of Scheinkman/LeBaron, cited by *Peters* (1991), p. 164.

[58] See *Peters* (1991), p. 164.

[59] See *Frank/Stengos* (1988), p. 125ff. Some researchers are so little interested in historical questions that they do not even give the running time of their time series in each of their publications.

of research began. In 1991 Hsieh again analyzed the CRSP US stock returns. This time he used different samples of weekly, daily and 15-minute prices, adjusted or nonadjusted for dividends. The author constructed different portfolios consisting of deciles ranked by size every quarter. Several measures such as the correlation dimension (calculated by the Grassberger-Procaccia algorithm), the BDS test and ARCH models led the author to reject IID, but he found conditional heteroskedasticity in his time series rather than chaotic behavior. With only 1.294-2.017 data points, his samples were very small.[60]

In the same year Peters published his book about 'Chaos and order in the Capital Markets'. It contains the analysis of several long running time series in financial markets. First he applied R/S analysis to the S&P 500, using monthly data from January 1950 until July 1988, estimating cycle lengths of 48 months. After a peak at $H = 0.78$, the Hurst exponent begins to fall, and soon follows the random walk line of $H = 0.50$. The cycles discovered in non periodic time series are characteristic of nonlinear dynamic systems. Peters did the same kind of analysis with individual stocks such as IBM, Mobil Oil, Coca Cola and Niagara Mohawk and had similar results. Tech stocks have slightly higher H and shorter cycle lengths, utilities lower H and longer cycle length than the index itself. This shows that risk reduction by diversification also works under chaotic modes. Investigating international stock markets with Hurst statistics, Peters used the Morgan Stanley Capital International (MSCI) index, ranging from January 1959 to February 1990. According to his results, the Hurst exponent of the UK and Japan were 0.68, Germany's H 0.72. Their cycle lengths differed between 30, 48, and 60 months. For 30 years monthly US T-bond yields from January 1950 until December 1989, Peters found an H 0.68 and a cycle length of 5 years, coincident with that of the US industrial production. On the other hand, during the same period, 3-, 6-, and 12-month T-bills showed a slightly lower H, but no cycle length was apparent in the log-log plot. This means that there was either not enough data, or no cycle length. With this consideration, T-bills are an extreme exception in that they show the unique character of money markets. Beginning with the end of the Bretton Woods system monthly currency rates of the years 1973 until 1990 have been analyzed, including FX rates of the US $ vs. ¥, £, and Sing(apour)$. While the first three rates show high levels of persistence with $H \approx 0.60$, the Sing$ is one of the few examples of a truly randomizing financial time series. While cycle lengths of the currency markets could not be identified exactly[61], several economic indicators of the US showed quite clear behavior. While the unadjusted time series (industrial production, and housing starts) had cycle lengths of about 5 years, composite index numbers (like the Department of

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[60] **See *Hsieh, David A.,* Chaos and Nonlinear Dynamics: Application to Financial Markets, in: *Journal of Finance* vol. XLVI, no. 5 (1991), p. 1.839-1.877, esp. p. 1.854-1.875.**

[61] **The $/£ and the $/DM seem to have cycle lengths of 6 years.**

Commerce Leading Economic Index or the Columbia University Leading Economic Index) had a shorter cycle length of 4.5 years. Hurst coefficients varied from 0.73 (housing starts) to 0.81 (new business formations) and 0.83 (Columbia Index) up to 0.91 (industrial production). This shows long memory effects even in economic data.[62]

In a next step, Peters showed some extraordinary stability in stock markets behavior over several decades. For this purpose, S&P 500 daily returns from January 2nd, 1928 until July 5th, 1990, covering 15.504 observations, have been investigated. The R/S analysis shows again cycle lengths between 900 and 1.000 trading days, or about 4 years. Dividing the data into 2.600 data points each covering about 10 years showed mean returns between -5.98 and +12.28% and standard deviations between 0.0993 and 0.3241. On the contrary, the Hurst coefficient showed astonishingly stable behavior. It only differed between 0.57 and 0.62, not at all mirroring the radically different environments: the Great Depression, three wars, the riotous 1960s, two oil shocks, the leverage boom of the 1980s, the stock market crashes of 1929, 1987 etc. This should be an interesting result for economic history (see fig. 8). Applying R/S analysis to stock market volatility, a very stable antipersistence with an H of 0.39 could be discovered, by analyzing monthly series of the standard deviation of daily returns from January 1945 to July 1990. Thus volatility is one of the very few antipersistent series found until now in economics. Peters' next step was to investigate daily inflation detrended stock returns in several countries: the S&P 500 from January 1950 to July 1989 and the MSCI Indices for Japan, Germany and the UK from January 1959 to February 1990 in local currency. He found fractal dimensions of 2.33, 3.05, 2.41, and 2.94. This means that three dynamic variables at a minimum are necessary to describe these national stock markets. This is in a striking contrast to Scheinkman/LeBaron's results, who found a fractal dimension of 5.7 in the US stock market[63] There might be several reasons for the difference. Peters applied the Wolf algorithm, while Scheinkman/LeBaron applied the Grassberger-Procaccia algorithm. Peters uses inflation detrended, but not dividend adjusted S&P 500 data, Scheinkman/ LeBaron use dividend adjusted, but not inflation detrended CRSP data. Both indices are partly composed of different stocks with different weights. Peters additionally points to the fact that for a fractal dimension of 610.400 = 10^6 data points would have been needed, whereas Scheinkman/LeBaron used only 5.000 observations. Whatever it is - the difference remains irritating and shows how much research needs to be done in that field. As a last step Peters determined Lyapunov exponents for monthly returns of the 4 national stock indices mentioned above. Peters computed a Lyapunov exponent for the US of L, - 0.0241. This means that the system loses predictive power after 1/0.024 ≅ 42 month's time. This is almost the cycle length of 4 years mentioned above. For

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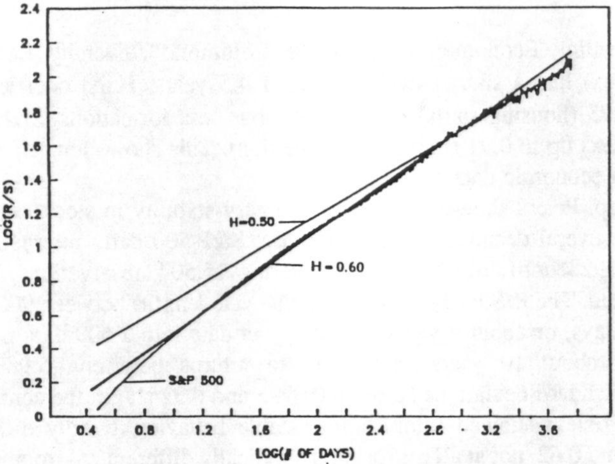
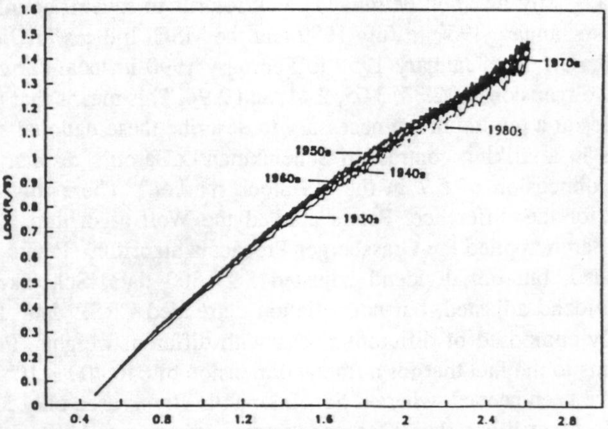
[62] **See *Peters* (1991), p. 84, 87-98.**

[63] **See an unpublished paper of Scheinkman/LeBaron, cited by *Peters* (1991), p. 164.**



R/S analysis: S&P 500 daily returns, January 2, 1928–December 31, 1989. Estimated H = 0.60. Note the cycle length of 1,000 trading days, or about four years, which we also saw in the monthly analysis in Figure 8.3.



R/S analysis: S&P 500 daily returns by decade. Note that the slopes do not change much from decade to decade.

Fig. 8: Stability of Hurst Exponent Source: Peters (1991), p. 111.

the UK, L_1 = 0.028 and a cycle length of 36 months has been found, for Japan L_1 = 0.0228 (44 months). For the German cycle length of 60 months, found via R/S analysis, about 50 years' data would be needed, which the MSCI index does not give.[64] Peters determines only the first and highest Lyapunov exponent L_1 but not the following two according to the system's fractal dimension. Loistl/Betz are right to criticize that this way Peters approximated correlation dimension could not be proved, e.g. by the Kaplan-Yorke dimensioa[65]

A deep, but narrow analysis of two German stocks (Commerzbank, Daimler) has been published by Holzer. He used daily Frankfurt closing prices from January 4th, 1983 until February 28th, 1992, each with 2.389 data points, prices adjusted for splits. In spite of the small data basis, the author applied a whole range of methods to the stocks, including spectral analysis, autocorrelation functions, the AR(2)-return maps, Poincaré sections and several residual tests. Applying the R/S analysis Holzer determined the Hurst exponents via Tukey's biweight estimator as H = 0.96 (Daimler) and H = 0.97 (Commerzbank). This points to strong long memory correlation.[66] The correlation dimension D = D(6) is 1.8 for Daimler and 2.2 for Commerzbank. This is extremely low, implying that only 2-3 substantial determinants might rule the systems. According to Brock[67] the double of D is a lead to distinguish the upper limit of numbers of these determinants. The residual test and the BDS test show evidence for the existence of deterministic chaotic dynamics in these stock prices. To determine the maximum Lyapunov exponent Holzer applied four different estimators (plateau, trend medium, Tukey, Huber). They give Ls between 0.00184 and 0.00301 (Daimler) and 0.00151 and 0.00300 (Commerzbank). All estimated Ls are positive.[68] Holzer also published one of the very few research papers done on commodity prices. He used 1.815 weekly weighted price averages for pig halves at Bavarian commodity exchanges from January 1957 until December 1991. He found a clearly lower Hurst exponent of 0.8295 (Huber estimator), or 0.8204 (Tukey biweight estimator), and an Lyapunov exponent L = L_1 = 0.006. His correlation dimension was D = D(8) = 3.6, meaning that 4 to 8 ($n_s = 8 \cong 2D + 1$) determinants are ruling the system. Holzer found that the two stocks were clearly ruled by less complicated and more steady modes. Nevertheless, both stock and commodity prices are governed by chaotic behavior.[69] Rather strong evidence of chaos was found by Alfredo Medio within a series of 4.204 observations of the DM/$ daily exchange rate from 1973 to 1989. His L converges to 0.12 and he figured out a correlation dimension of $D \cong 2.11$.[70] A

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[64] **See ibid., p. 110-112, 118, 164, 168-180.**
[65] **See *Loistl/Betz* (1993), S. 107.**
[66] **See *Holzer, Christian,* Analyse empirischer Datenreihen in der Ökonomie mit Instrumenten der nichtlinearen Dynamik. Diss. TU München, München 1992, S. 126.**
[67] **See *Brock* (1986), p. 176.**
[68] **See *Holzer* (1992), S. 114-137.**
[69] **See ibid., S. 138-152.**
[70] **See *Medio* (1992), p. 274-282.**

year ago, Blank detected chaos in future prices. He used the standard contracts of S&P 500 and Soya beans.[71]

At the end of this chapter, some particular historical work should be mentioned, done by D.A. Peel et al. about forex markets during German galloping and hyper-inflation. The data basis were 824 daily observations of intra day high rates and 140 weekly observations of closing rates, both on the London £/M spot market between January 2rd/8th 1921 until September 8th 1923. By the help of an ARCH model, BDS statistics and several other tests, the authors rejected IID. They estimated correlation dimensions between 4 and 5, suggestive that the rates were generated by a low dimensional process. On their very small data basis, they gave estimations of the time series' Lyapunov exponents finding that the hypothesis could not be rejected that the regarded forex rates exhibited chaotic behavior.[72]

Chaos research on financial markets until now mainly covers stock, bond and forex markets in several important countries during periods beginning with the 1950s, but there are also papers concerning previous periods and selected commodity prices, derivatives and money markets. This short resumé already shows that this research points out the existence of nonlinearities, and in most cases even of chaos.

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[71] **See *Blank, S.C.,* "Chaos" in Future Markets? A Nonlinear Dynamic Analysis, in: *The Journal of Future Markets* vol. 11, no. 6, p. 711-728. Out of a number of further publications see for the stock markets: *Funke, Michael,* Testing for Nonlinearity in Daily German Stock Returns. (Discussion Paper 30-93, FU Berlin and Centre for Economic Forecasting, London Business School) Berlin and London; *Philippatos, George C,* Instabilities and Chaotic Behavior of Stock Prices in International Capital Markets, in: *Managerial Finance* vol. 20, no. 5/6, p. 14-42; *Tata, Fidelio* and *Vassilicos, Christos J.,* Chaos in the Stock and Forex Markets? (Discussion Paper no. 64, Forschungsgemeinschaft für Nationalökonomie an der Hochschule St. Gallen) St. Gallen 1992; *Willey, Thomas,* Testing for Nonlinear Dependence in Daily Stock Indices, in: *Journal of Economics and Business* vol. 44, no. 1 (1992), p. 63-76; for the bond markets: *Larrain, Maurice R.,* Testing Chaos and Nonlinearities in T-Bill Rates, in: *Financial Analysts Journal* vol. 47, no. 5 (1991), p. 51-62; for the FX markets: *Aczel, Amir D.* and *Josephy, Norman H.,* The Chaotic Behavior of Foreign Exchange Rates, in: *The American Economist* vol. 35, no. 2 (1991), p. 16-24; *Müller, Ulrich A.* et al., Statistical Study of Foreign Exchange Rates, Empirical Evidence of a Price Change Scaling Law, and intraday Analysis, in: *Journal of Banking and Finance* 14 (1990), p. 1.189-1.208.**

[72] **See *Peel, D.A.* and *Yadav, P.,* The Time Series Behaviour of Spot Exchange Rates in the German Hyper-Inflation Period: Was the Process Chaotic? in: *Empirical Economics* 20 (1995), p. 455-471; *Peel, DA.* and *Speight, A.,* Testing for Non-Linear Dependence in Inter-war Exchange rates, in: *Weltwirtschaftliches Archiv* 130 (2) (1994), S. 391-417.**

Applications of the chaos theory to economic history and to historical financial markets

Application of nonlinear mathematics: The measures described above may be applied to historical financial markets, but until now only a few periods of time, countries and markets have been studied. One specific problem is the lack of long time series collected and handled in the same way. In many countries within the last 200 years several adjustments are required. E.g. if currency reforms took place or quotation technics changed from percentage quotation to unit quotation. In some cases, indices have been calculated, but with changes in method during the course of time. The data have to be investigated and made comparable, when for instance a stock price index was changed into a performance index, small caps were included into a clear blue chip index, a value weighted index replaced an unweighted index, or an index first uses spot prices, then average prices and later closing prices. In Germany, for example, there are five long periods of continuous monthly stock price indices: for the years 1870-89, 1890-1913, and 1914-24, from January 1924 until June 1943, and from January 1950 until April 1995[73]. Each of these periods of German stock index include different numbers of stocks. The oldest one is not weighted and without capital adjustments, the newer ones used different methods. Nevertheless the Federal Statistic Bureau put the first three periods together. In addition to these problems, there are gaps to be filled such as the one from August 1914 until October 1917, in August and from October 1931 until March 1932, and from July 1943 until December 1949. Even a much broader and more interesting gap in monthly data for German stock indices reaches from the market's beginning at the end of the year 1835 and extends until December 1869.[74]

Equally difficult is the work to be done to get acceptable bond market data. Until recent times, no performance indices have been calculated and published in Germany. Only average prices, distinguished by different coupons were available. Until World War II, the bond rate was calculated as if all the bonds were perpetuities. To reconstruct an adjusted bond index, loan terms of each bond have to be investigated. Similar problems are typical for money market prices, mainly resulting from changing market segments. An ambitious DFG project is reconstructing German stock and bond markets in imperial times.[75]

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[73] The Statistisches Bundesamt stopped calculating its stock market index, adjusted for splits, but not for dividends.

[74] The author of this articel prepares a summary of several German stock prices and indices of the time between 1835 and 1870.

[75] Bond section has been published and is available for research. See *Müller, Johannes,* Der deutsche Rentenmarkt vor dem Ersten Weltkrieg - eine Indexanalyse. (Schriftenreihe des Instituts für Kapitalmarktforschung an der Universität Frankfurt/M. Bd. XV) Diss. Frankfurt/M., Knapp, Frankfurt/M. 1992.

For foreign exchange and currency markets a broad investigation of prices and cross rates already has been published.[76] Nevertheless these few items show how much needs to be done to get sufficient data for analyzing certain financial markets for chaotic behavior. For only a few countries including the US and the UK there are almost continous financial market data available.

Another problem is the pure quantity of data needed, even on financial markets. Only a few market sections are able to generate time series with a magnitude of data points large enough to apply methods of chaos theory to it. Although some recent investigations for R/S analysis used data pools of only about 500 observations to calculate Hurst coefficients and long memory cycles, usually 2-3.000 observations are needed for these methods. Rules of thumb developed by chaos researchers in physics state that 30^D data points are necessary for calculating a system's Lyapunov exponent In the case of a fractal dimension of $D = 3$ this would mean 27.000 data points; 810.000 observations for D = 4 would be necessary. Recent work showed that with the help of certain algorithms, L could be calculated on the basis of only 3.000 data points with sufficient accuracy. Even for 2-3.000 data points periods of 5.5-10 years are necessary, dependant on the number of trading days per year. If only monthly data are available, for 2.000 observations a period of 166.6 years would be needed. After 200-500 data points only a general statement about the algebraic sign of L is possible.[77]. These few examples show how narrow the application is with these methods even in the field of historical financial markets.

Nevertheless there are some possible and promising applications to historical data sets. As a first step analyzing for nonlinearities is useful in most cases. If this analysis results in rejecting IID, methods relying on it such as standard deviation etc. could no longer be applied. In these cases a recently discussed alternative is to use the fractal dimension D as a measure of volatility and risk.[78] R/S analysis could be another method for investigating historical financial time series. Calculating the Hurst coefficient would detect the kind of process which determines a specific market at a specific time. Chaotic methods could give hints for long memory cycles, and, if there are enough data, even of the time at which the system's mode changes. Even if it would not be possible to identify a market's determinants, it would be very useful to know how many of them are working in the system. Knowing the times of mode changes could give hints

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[76] **See *Spufford, P.,* Handbook of Medieval Exchange. London 1986; *McCusker, J J.,* Money and Exchange in Europe and America, 1600-1775. A Handbook. Chapel Hill 1978; *Schneider, Jürgen* et at, Währungen der Welt I-X. Franz Steiner, Stuttgart 1991-1994.**

[77] **See *LoistllBeli* (1993), S. 56-64; *Seifritz, Walter,* Wachstum, Rückkoppelung und Chaos. Hanser, München u. Wien 1987, S. 58-61; *Steeb, Willi-Hans* u. *Kunick, Albrecht,* Chaos in dynamischen Systemen, 2. Aufl., BI Wissenschaftsverlag, Mannheim etc. 1989, S. 41-48; *Buzug* (1994), S. 37-55; *Steeb* (1991), p. 87-88. For several algorithms for approximating *L* see *LoistllBetz* (1993), S. 70-80.**

[78] **See Peters (1991), p. 59f.**

for the reasons causing these changes. Information like this could even be useful for today's discussion about the stability of financial markets. This is especially true in the case of derivatives like options and financial futures being repressed in the 1890s and after the stock market crash of 1929 with the argument of destabilizing outright markets.

Causality, unpredictability and scale invariance: Until now, however, the main importance of chaos theory does not lie in its mathematical application. The overwhelming number of research so far done points out, that financial markets and macroeconomic time series in several countries during several periods show chaotic behavior. This means that they follow certain rules that could generally be important for financial or even economic history. One of the most important rules is the sensitive dependence on initial conditions (fig. 9 shows the behavior of such a system). Economic and technical historians know this effect very well in several fields. It is one of the reasons why in many cases it is so difficult to find out why certain inventions succeed over competing ones. A well known example out of recent times is the success story of VHS against two competing video technics even though its technical standard was inferior. Its advantage was a market start only a few months earlier. Another perhaps still more famous example is the question why in late 19th and beginning 20th century the internal combustion engine succeeded in competition with light steam and electrical engines as the motor for automobiles. This process is still in discussion showing that it is often very difficult to decide for what reasons a certain process took a certain direction.[79] Similar situations are well known from political history especially during pre-war crises, when psychological effects such as group dynamics or behavior under stress do play a dominant role. Most processes in history, however have almost an endless prehistory. Looking at a process already in motion or even everlasting, the principle of sensitive dependence on initial conditions has to be regarded in general as a strong sensitivity to small changes in parameters.[80] The history of innovations is only a very striking example for this kind of sensitivity. Some authors even believe that deterministic processes are interrupted from time to time, and history systematically creates unpredictable events which have important long term effects.[81]

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[79] Brian Arthur discussed a whole range of such situations in his article: *Arthur, W. Brian,* Self-reinforcing mechanisms in economics, in: *Anderson, P.W., KJ. Arrow* and *D. Pines* (eds.): The Economy as an Evolving Complex System. Addison-Wesley, Redwood City, CA, 1988, p. 9-31.

[80] In deterministic chaotic systems this may be described as sensitivity of eigenvalues. See *Steeb* (1991), p. 161.

[81] See *Ruelle, David,* Chance and Chaos. Princeton University Press, Princeton, N.J., 1991, chapter 14 (Historical Developements).

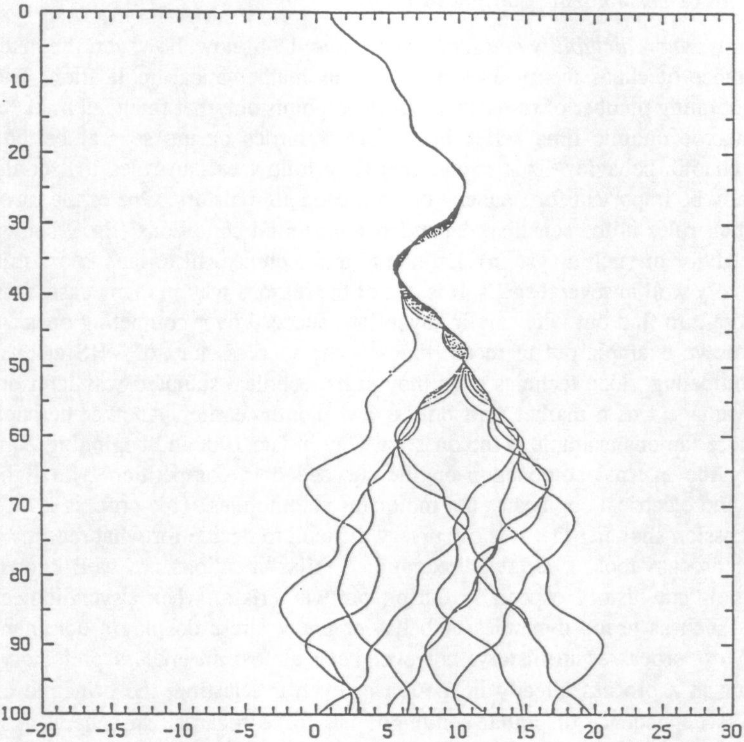

Fig. 9: Sensitive Dependence on Initial Conditions Source: Tvede (1991), p. 62.

Under such circumstances sciences have a particular problem, if they try to be exact in a mathematical sense: Even with the help of perfect methods, only one of a system's determinants can be measured with endless exactness. This law, known as Heisenberg's uncertainty principle was first demanded for quantum mechanics, but today is thought to be one of the few universal scientific laws. This means that there is a hindrance in principle for analyzing historical markets in a perfect manner. The uncertainty principle is so to speak the mathematical proof that by the help of mathematics, we are not able to analyze the economic behavior of a market as a whole. Nevertheless we are able to calculate at least individual characteristics separately as shown above. Both, the butterfly effect and the uncertainty principle, cause a kind of hare/ hedgehog-problem, which means that it often does not make sense to look too exactly for concrete reasons for a certain development. It is more sensible to try to identify the inner dynamics of the process instead. We will follow that later on, when we look at the advantages that psychological theories might have in the investigation of historical financial markets.

Another important consequence of chaos in financial markets comes from their unpredictability after a certain period of time. After this period of time the strong causality principle has no more validity.[82] That means arguments by analogy are not possible. Historians know this very well saying 'history does not repeat itself. But this is not the whole truth. During this period of time, the strong causality principle is certainly valid. This is already a self-evident fact in the historian's work every day, but it is worth while looking at this rule of thumb from a more formal point of view. In addition, most chaotic markets include deterministic structure. One of the outcomes of this determinism is scale invariance. For one thing this means that it does not matter which time scale is chosen for analysis. This might be of importance for economic historians working in a quantitative way, because in certain situations scale invariance might simplify their data problem. This is especially true for financial markets, where it does not matter whether prices, turnover or returns are minutely, hourly, daily, or even monthly. Scale invariance also means self similarity: In the course of time similar patterns in similar situations could be found.

Endogenous vs. exogenous reasons for sudden changes of a system's behavior: Nonlinear dynamic systems may enter chaotic phases because of exogenous or endogenous reasons. That means that financial markets and other economic and social systems do not need external shocks to show erratic behavior, if they are chaotic systems. Endogenous determinants are not necessarily only those representing market organization and psychology of market operators. They might also be economic determinants, if they were only part of the variables usually determining the system.[83] As we do not yet know for any financial

[82] **Similar causes give similar results.**

[83] **That is variables necessary for describing the system in a mathematical sence.**

market analyzed so far which these determinants are, it is still an unanswered question, whether economic or organizational and psychological determinants are more important for financial markets. However, there is a slight tendency in favor of the last ones. As we know from recent research, some chaotic measures did not change much over several decades. This is also true for market organization and according to contemporary market comments for the operators' psychology, but not for a whole range of economic indicators. The 'inner surroundings' of a specific financial market,[84] are sufficient to explain the system's behavior during normal times. On the other hand, these systems refer to a certain stability even outside the core fundamental data. If they reach specific thresholds, determinants from the market's 'outer surroundings' become necessary to explain its movements, such as unemployment, strikes, political elections etc. This is usually a time when financial markets show erratic movements. This could be seen very well during the period of Weimar Germany, when many of these determinants fluctuated much more than in the times before and after, with the result of erratic price movements on financial markets (see fig. 10).

The quality of chaotic systems, even if they show deterministic behavior, can hardly be described by common closed economic models, because these models usually assume that the system in question tends to an equilibrium that is exactly determined. Only very few of these models suppose that such a system tends to oscillate within a certain range. In other words: they assume that the system has either a point attractor or a cyclical attractor. The first type of model cannot even describe a deterministic chaotic system during the non-chaotic mode, when a nonlinear dynamic system oscillates a certain time within a certain range.[85] After the next bifurcation, the second kind of closed economic model also loses its explaining power. As a further problem deviations from that equilibrium in closed economic models are only possible by external shocks, but in chaotic systems endogenous reasons might be even more important. Nevertheless especially for analyzing macroeconomic problems in a formal way, there is often no alternative to these models, but when using them, their limits should be kept in mind.[86]

One way to treat this dilemma may be to model the chaotic behavior of financial markets.[87] This could include some psychological theories, since the

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[84] **See *Luhmann, Niklas,* Die Wirtschaft der Gesellschaft. Suhrkamp, 2. Aufl., Frankfurt/M. 1989, S. 116-118.**

[85] **With a limit-cycle as an attractor.**

[86] **Aside business cycles and financial markets, closed economic models tending to an equilibrium in many cases at least on the short run imitates reality in a sufficient way.**

[87] **See *Lorenz* (1987), p. 25-30. Besides these problems, these models have a severe problem concerning the general philosophy of science, if they should be tested empirically: They cannot be disproved because possible external shocks in most cases cannot be named and therefore are only considered by a general ceteris paribus proviso. That's why they do not fulfill Popper's criterion. See *Raffée, Hans,***

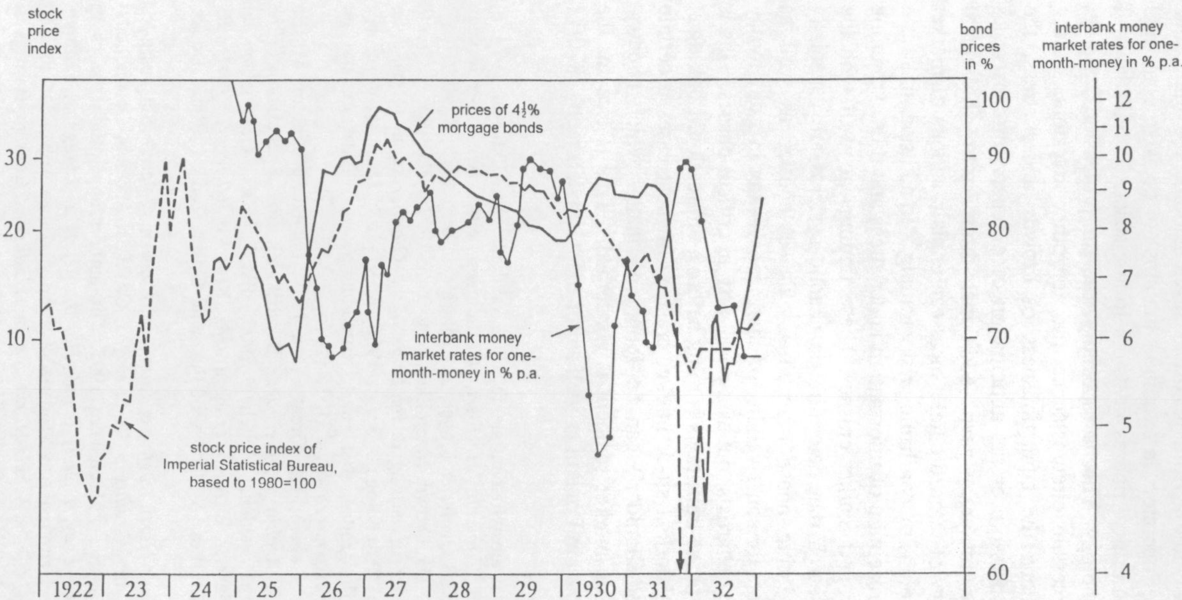

Fig. 10: Weimar Germany Source: Kiehling (1991), p. 114.

situation during the critical phase transitions corresponds in a better way with these theories.[88] As we know from historical investigations, the immediate surroundings of market operators in extreme situations are decisive compared to economic facts.[89] In critical phases investors follow strictly certain laws of crowd or group psychology. This is especially true for professional market operators, but under certain circumstances also for other investors. During hectic market operations the contagiousness of crowds under stress, the importance of group pressure or the anticipation of a supervisor's becomes predominant. Recent research gave additional hints from individual psychology. The theory of learned carelessness can explain actions of market operators in critical months by confirming risky trading.[90] In addition, there is a gating effect limiting reception of information under stress and thus explaining short run behavior around market crashes. These findings go well with the findings of chaos research that deterministic chaotic systems are especially sensitive to shocks during phase transitions.[91] These theories as well as self-similarity give hints for some validity of chart theory - not in a day to day manner as it is used by chartists, but as a possibility in critical phases. This is especially true during phase transitions coming before a bifurcation. Looking at the situation before historical stock market crashes, in most cases a certain chart formation ('head-shoulder') can be found, no matter which country, which century or which market segment is concerned.[92] This is true for the Southsea Bubble of 1720 in London as well as for the 'Gründerkrach' 1870/71

Grundprobleme der Betriebswirtschaftslehre. UTB Vandenhoeck, Göttingen 1974, S. 34; *Prim, Rolf* and *Tilmann, Heribert,* Grundlagen einer kritisch-rationalen Sozialwissenschaft. UTB Quelle & Meyer, 2. Aufl., Heidelberg 1975, S. 70.

[88] **See *Le Bon, Gustave,* The Crowd. Macmillan, New York 1922; *Seidenfus, Hellmuth Stefan,* Zur Theorie der Erwartungen, in: *Schmölders, Günther* et al. (Hg.), John Maynard Keynes als "Psychologe". Berlin 1956, S. 97-158; *Dinauer, Josef W.,* Psychologische Einflußgrößen bei der Kursbildung am Aktienmarkt, in: *DVFA-Beiträge zur Wertpapieranalyse* H. 15 (1976).**

[89] **See *Galbraith, John Kenneth,* The Great Crash 1929. 6th ed., Houghton Mifflin Comp., Boston 1988; *Aschinger, Gerhard,* Börsenkrach und Spekulation: Eine ökonomische Analyse. Vahlen, München 1995; *Wirth, Max,* Geschichte der Handelskrisen. Sauerländer, 2. Aufl., Frankfurt/M. 1874; *Kindleberger, Charles P.,* Manias, Panics and Crashes: A History of Financial Crises. 2nd ed., Basic Books, New York 1989.**

[90] **See *Frey, Dieter, Schulz-Hardt, Stefan* u. *Lüthgens, Carsten,* Gelernte Sorglosigkeit und Risikoakzeptanz, in: *Wenninger, G.* u. *Hoyos, C.* (Hg.), Arbeits-, Gesundheits- und Umweltschutz. Asanger, Heidelberg 1995. The authors apply their theory to financial markets. See *Frey, Dieter, Schulz-Hardt, Stefan* u. *Lüthgens, Carsten,* Termingeschäfte: Das Hauptrisiko bei Finanzderivaten sind ihre Anwender, in: *Zeit* 40 (1994), S. 33; *Frey, Dieter, Schulz-Hardt, Stefan* u. *Lüthgens, Carsten,* Barings ist überall, in: *Woche* v. 17.3.1995, S. 20f. Until now no specific scientific paper has been published.**

[91] **See *Shaffer* (1991), p. 209-212.**

[92] **Roughly 3/4 show socalled head-sholder formations.**

in Berlin, the 1929 stock market crash in New York, or the crashes of 1962 in Frankfurt and of 1987 in dozens of market places around the world.

Continuity vs. singularity: In deterministic chaotic systems, there is an inner coherence between their continuity and the singularity of certain events. On the one hand, the butterfly effect causes unpredictability. Poincaré recurrence is not possible, if such a system is quasi-periodic.[93] On the other hand, values of Hurst coefficient for most financial markets point to a long run correlation causing long memory cycles and the Joseph effect This expression refers to the biblical story of Joseph who predicted seven fat years followed by seven lean years. It characterizes the phenomenon that certain events are significantly more frequent than they should be according to random laws, e.g. stock market crashes, and are probably caused by a bifurcation. This leptokurtosis found in changes of time series is regarded as a first sign of its chaos. In a system characterized by intermittency during some phases the strong causality principle is valid, while in other phases only the weak causality principle[94] is valid. Continuity and singularity of events are qualities of the same developments, no matter whether the systems show bifurcations or intermittency. Existing social and economic systems usually do not follow these models exactly. In history we rather may distinguish between phases in which analog conclusions are possible in a more or less extended period.

As we have seen above, long-run correlations and superior control loops are not the only elements of continuity within deterministic chaotic systems. They also show similar behavior during critical periods such as phase transitions. Knowing that at critical times and under comparable circumstances similar situations in history took place again and again because of similar psychological conditions, maybe it is even time to correct our model of history as not repeating. Maybe the idea of 'archipelagos of recognition' giving form to unstructured surroundings is more helpful. These critical points are often branch points for further development and are therefore of extreme importance. The identification and clear description of such 'archipelagos' would also be helpful for recognizing, which kind of a system's condition is transferred from the past to the present or to the future. This is even more true if we are able to combine these considerations with the short run predictability of deterministic chaotic systems. According to the above definition chaotic systems show that some global characteristics do not react sensitively to initial conditions. These global characteristics may be interpreted as superior control loops.[95] Regarding all this, the traditional contrast between the idea of continuity in teleological conceptions of history[96] and the idea of singularity in historicism[97] loses its purpose in deterministic chaotic systems.

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[93] See *Steeb* (1991), p. 11 ff.

[94] Equal causes bring equal results.

[95] It might be undecided whether such loops can be proved. To do so could be the field of historical economists, if the reflections about closed economic models do not oppose.

[96] See *Spengler, Oswald,* Der Untergang des Abendlandes. 6. Aufl., Beck, München

To look at two specific financial markets make these thoughts more clear. 'Archipelagos of recognition' could be found around markets crashes. To find superior control loops one has to look deeper. According to H.C. Zeeman's model of the stock market as a cusp catastrophe, equity markets are determined by two groups of investors, their motivation, and certain feedback mechanisms of their action. This model is very consistent with reality over a long period of time, that is ever since modem financial markets were formed in the 18th and 19th century.[98] On the other hand market qualities such as velocity of reaction, legal framework, rules and regulations, entry conditions, international coherence, and some other criteria changed markedly. Even these criteria were constant to some extent, but from time to time there were also considerable changes. The actual combination of these criteria determines long memory cycles. The changes of specific criteria usually took place very rapidly. E.g. valuation of German stocks has been done over 120 years by comparing their yield to that of the bond market. In the beginning of the 1960s this kind of comparison changed within a few months for the price/earnings ratio. Examples from the same market Sales figures grew in a step curve with steps at the beginning of second German Empire, around 1960 and in the 1980s. Derivatives boomed during the 1880s and again from the 1980s. Legal and organizational framework changed dramatically with the Prussian stock company law of 1843, the General German Commercial Code (ADHGB) at the beginning of the 1860s, the Amendment of German Stock Market Law in 1896, and the Second Law for the Promotion of Financial Markets in 1994.

German bond market followed a similar, but slightly different pattern. Over long periods of time, it is characterized by relatively small price oscillations. The ratio of highest to lowest yields was at best 3:1. Except for some short periods, e.g. 1922/23, no or very few speculative investors could be found between 1835 and 1980." During the last 100 years it was common to pay back bonds at par at the end of their maturity period. In addition a small number of determinants usually makes market forecasts easier: the spread to money markets, between market segments and to international markets, tax reasons etc. On the other hand, the German bond market has always been affected by severe fractions. Unlike stock markets whole bond market segments have been eliminated every 30 to 60 years: in the course of national bankruptcies in Napoleonic times, the American railway crash 1872/73, and (much more

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1980, S. 3. An extreme version gives Baur deviding up history into oscillations of mentality and art history. See *Baur, Karl,* Zeitgeist und Geschichte: Versuch einer Deutung. Callwey, München 1978.

[97] **See *Seiffert, Helmut,* Wissenschaftstheorie** 2. **Beck, 9. Aufl., München 1991, S. 63-69.**

[98] **See *Kiehling, Hartmut,* Kursstürze am Aktienmarkt, dtv, München 1991.**

[99] **Since the beginning of the 1980s, German government bonds are one of the vehicles for FX speculation with the D-Mark, because of the lack of heavily traded money market papers.**

evident) after World War I and II. In addition, the rise of new types of securities was of much greater evidence in clear German bond markets than in complicated stock markets. Such an 'innovation' was the abundance of the creditor's redemption in late 19th century. From time to time, the government tends to make interventions in its own favor e.g. tax sheltered Bunds in the 1950s; the privilege of eligibility for trusts, which railway securities received 1843, and much safer mortgage bonds usually not before the 1890s.

Because of the importance of the hermeneutic method for historical research in German speaking countries a few more words should be added concerning this method.[100] If it is taken seriously, there can be no laws in history.[101] An opposite opinion has been shown by historians from other countries. In France, researchers including Fernand Braudel 'á la longue durée' showed laws, superior control loops (or whatever we should call them).[102] Several Anglosaxon historians were open to long term considerations in a similar manner. A well known example was Paul Kennedy's 'The Rise and the Fall of Great Powers' of 1987, to which on the basis of 500 years of economic, military and political evolution, he added a chapter predicting the likely development in the 21th century.[103] Researchers using the hermeneutic method approve of the existence of analogies, parallelisms and similarities in history, but they emphasize that these phenomenon are not laws in the sense of bases of extended spatial and temporal developments.[104] A general suspicion of these historians against these phenomenon as well as against the generalization of (psychological or other) types remains unchanged. This discussion today is an open one, at least many questions cannot be decided on the help of our present day knowledge, but the above thoughts might give ideas for discussion.

Resume: Chaotic systems like most financial markets show the paradox that they cannot be explained or predicted in spite of the fact that they show surprisingly constant behavior to some extent. Despite of that paradox or even because of it, nonlinear dynamics and chaos theory might be useful for economic historians. Some of the above conclusions and possible applications may sound unusual to an historian, while others have been known for a long

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100 See *Bichler, Reinhold,* Das Diktum von der historischen Singularität und der Anspruch des historischen Vergleichs: Bemerkungen zum Thema Individuelles versus Allgemeines und zur langen Geschichte deutschen Historikerstreits, in: *Acham, Karl* u. *Schulze, Winfried* QHg.): Teil und Ganzes. Theorie der Geschichte: Beiträge zur Historik, Bd. 6, dtv, München 1990, S. 169-192; *Geldsetzer, Lutz,* Art. Hermeneutik, in: *Seiffert, Helmut* u. *Radnitzky, Gerard,* Handlexikon zur Wissenschaftstheorie, dtv, 2. Aufl., München 1994, S. 127-139.

101 See *Seiffert* (1991), S. 176f.

102 See *Braudel, Fernand,* Sozialgeschichte des 15.-18. Jahrhunderts, Bd. 3: Aufbruch zur Weltwirtschaft. Kindler, München 1986, S. 73-92, 327-346.

103 See *Kennedy, Paul,* The Rise and Fall of the Great Powers. Random House, New York 1987.

104 See Seiffert, 2, S. 178.

time. Even if this is the case for most of what has been said within the last section, chaos theory might cause a step forward if it is able to explain common effects in a formal way, and thus even give ideas for a formally orientated, but flexible theory of long term development of financial markets (and maybe even other systems in history). On the other hand, it is good to know that chaos theory confirms some important rules of historical research such as admissibility of analogies only within narrow temporal and functionally limits, to give just one example. This article necessarily only gives a survey and an incomplete one at that. This is true because of lack of space and because of the fact that the investigation of financial markets via chaos theory still stands at its beginning. Nevertheless a discussion of some of these questions might already be profitable. If this articles contains something reaching beyond plain explanation of chaos theory and research, this is maybe pleading for methodological openness in historical research.

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